Questions

Q1.

Given that

$$y = \frac{3\sin\theta}{2\sin\theta + 2\cos\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

show that

$$\frac{dy}{d\theta} = \frac{A}{1 + \sin 2\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

where A is a rational constant to be found.

(5)

(Total for question = 5 marks)

Q2.

A scientist is studying a population of mice on an island.

The number of mice, N, in the population, t months after the start of the study, is modelled by the equation

$$N = \frac{900}{3 + 7e^{-0.25t}}, \quad t \in \mathbb{R}, \quad t \geqslant 0$$

(a) Find the number of mice in the population at the start of the study.

(1)

(b) Show that the rate of growth $\frac{dN}{dt}$ is given by $\frac{dN}{dt} = \frac{N(300 - N)}{1200}$

(4)

The rate of growth is a maximum after T months.

(c) Find, according to the model, the value of T.

(4)

According to the model, the maximum number of mice on the island is P.

(d) State the value of P.

(1)

(Total for question = 10 marks)

Q3.

$$y = \frac{5x^2 + 10x}{(x+1)^2} \qquad x \neq -1$$

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{A}{(x+1)^n}$ where A and n are constants to be found.

(4)

(b) Hence deduce the range of values for x for which $\frac{\mathrm{d}y}{\mathrm{d}x} < 0$

(1)

(Total for question = 5 marks)

Q4.

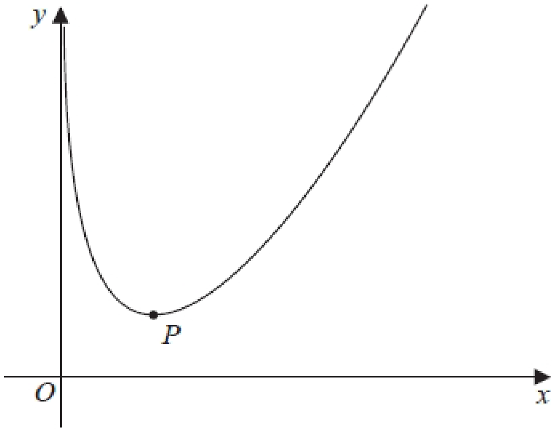

Figure 1

Figure 1 shows a sketch of the curve *C* with equation

$$y = \frac{4x^2 + x}{2\sqrt{x}} - 4\ln x \qquad x > 0$$

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}}$$

(4)

The point *P*, shown in Figure 1, is the minimum turning point on *C*.

(b) Show that the *x* coordinate of *P* is a solution of

$$x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}$$

(3)

(c) Use the iteration formula

$$x_{n+1} = \left(\frac{4}{3} - \frac{\sqrt{x_n}}{12}\right)^{\frac{2}{3}} \qquad \text{with } x_1 = 2$$

to find (i) the value of x_2 to 5 decimal places,
(ii) the *x* coordinate of *P* to 5 decimal places.

(3)

(Total for question = 10 marks)

Q5.

The function g is defined by

$$g(x) = \frac{3\ln(x) - 7}{\ln(x) - 2} \qquad x > 0 \qquad x \neq k$$

where *k* is a constant.

(a) Deduce the value of *k*.

(1)

(b) Prove that

$$g'(x) > 0$$

for all values of *x* in the domain of g.

(3)

(c) Find the range of values of *a* for which

$$g(a) > 0$$

(2)

(Total for question = 6 marks)

Q6.

Given that

$$y = \frac{x - 4}{2 + \sqrt{x}} \qquad x > 0$$

show that

$$\frac{dy}{dx} = \frac{1}{A\sqrt{x}} \qquad x > 0$$

where *A* is a constant to be found.

(Total for question = 4 marks)

Q7.

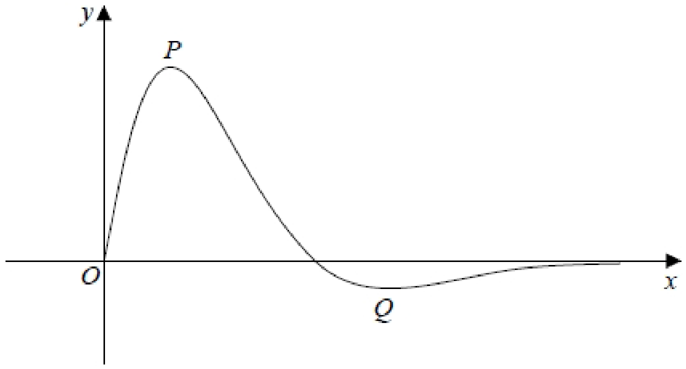

Figure 5

Figure 5 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{4\sin 2x}{e^{\sqrt{2}x-1}}, \quad 0 \leqslant x \leqslant \pi$$

The curve has a maximum turning point at P and a minimum turning point at Q as shown in Figure 5.

(a) Show that the x coordinates of point P and point Q are solutions of the equation

$$\tan 2x = \sqrt{2}$$

(4)

(b) Using your answer to part (a), find the x-coordinate of the minimum turning point on the curve with equation

(i) $y = \mathrm{f}(2x)$.
(ii) $y = 3 - 2\mathrm{f}(x)$.

(4)

(Total for question = 8 marks)

Q8.

Given $y = x(2x + 1)^4$, show that

$$\frac{dy}{dx} = (2x+1)^n(Ax+B)$$

where n, A and B are constants to be found.

(4)

(Total for question = 4 marks)

Q9.

$$\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv A + \frac{B}{(x-3)} + \frac{C}{(1-2x)}$$

(a) Find the values of the constants *A*, *B* and *C*.

(4)

$$f(x) = \frac{1+11x-6x^2}{(x-3)(1-2x)} \qquad x > 3$$

(b) Prove that f (x) is a decreasing function.

(3)

(Total for question = 7 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
	$\frac{dy}{d\theta}=\frac{(2\sin\theta+2\cos\theta)3\cos\theta-3\sin\theta(2\cos\theta-2\sin\theta)}{(2\sin\theta+2\cos\theta)^2}$	M1 A1	1.1b 1.1b
	Expands and uses $\sin^2\theta+\cos^2\theta=1$ at least once in the numerator or the denominator or uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\frac{dy}{d\theta}=\frac{...}{.........C\sin\theta\cos\theta}$	M1	3.1a
	Expands and uses $\sin^2\theta+\cos^2\theta=1$ the numerator and the denominator AND uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\frac{dy}{d\theta}=\frac{P}{Q+R\sin 2\theta}$	M1	2.1
	$\Rightarrow\frac{dy}{d\theta}=\frac{3}{2+2\sin 2\theta}=\frac{3/2}{1+\sin 2\theta}$	A1	1.1b
		(5 marks)	

Notes:

M1: For choosing either the quotient, product rule or implicit differentiation and applying it to the given function. Look for the correct form of $\frac{dy}{d\theta}$ (condone it being stated as $\frac{dy}{dx}$) but tolerate slips on the coefficients and also condone $\frac{d(\sin\theta)}{d\theta}=\pm\cos\theta$ and $\frac{d(\cos\theta)}{d\theta}=\pm\sin\theta$

For quotient rule look for $\frac{dy}{d\theta}=\frac{(2\sin\theta+2\cos\theta)\times\pm...\cos\theta-3\sin\theta(\pm...\cos\theta\pm...\sin\theta)}{(2\sin\theta+2\cos\theta)^2}$

For product rule look for

$\frac{dy}{d\theta}=(2\sin\theta+2\cos\theta)^{-1}\times\pm...\cos\theta\pm3\sin\theta\times(2\sin\theta+2\cos\theta)^{-2}\times(\pm...\cos\theta\pm...\sin\theta)$

Implicit differentiation look for $(...\cos\theta\pm...\sin\theta)y+(2\sin\theta+2\cos\theta)\frac{dy}{d\theta}=...\cos\theta$

A1: A correct expression involving $\frac{dy}{d\theta}$ condoning it appearing as $\frac{dy}{dx}$

M1: Expands and uses $\sin^2\theta+\cos^2\theta=1$ at least once in the numerator or the denominator OR uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\frac{dy}{d\theta}=\frac{...}{.........C\sin\theta\cos\theta}$

M1: Expands and uses $\sin^2\theta+\cos^2\theta=1$ in the numerator and the denominator AND uses $2\sin\theta\cos\theta=\sin 2\theta$ in the denominator to reach an expression of the form $\frac{dy}{d\theta}=\frac{P}{Q+R\sin 2\theta}$.

A1: Fully correct proof with $A=\frac{3}{2}$ stated but allow for example $\frac{3/2}{1+\sin 2\theta}$

Allow recovery from missing brackets. Condone notation slips. This is not a given answer

Q2.

Question	Scheme	Marks	AOs
	$N=\dfrac{900}{3+7\mathrm{e}^{-0.25t}}=900(3+7\mathrm{e}^{-0.25t})^{-1},\ t\in\mathbb{R},\ t\geq 0\ ;\quad \dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{N(300-N)}{1200}$		
(a)	90	B1	3.4
		(1)	
(b) Way 1	$\dfrac{\mathrm{d}N}{\mathrm{d}t}=-900(3+7\mathrm{e}^{-0.25t})^{-2}\left(7(-0.25)\mathrm{e}^{-0.25t}\right)\ \left\{=\dfrac{900(0.25)(7)\mathrm{e}^{-0.25t}}{(3+7\mathrm{e}^{-0.25t})^2}\right\}$	M1	2.1
		A1	1.1b
	$\Rightarrow \dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{900(0.25)\left(\left(\dfrac{900}{N}-3\right)\right)}{\left(\dfrac{900}{N}\right)^2}$	dM1	2.1
	correct algebra leading to $\dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{N(300-N)}{1200}$ *	A1*	1.1b
		(4)	
(b) Way 2	$\dfrac{\mathrm{d}N}{\mathrm{d}t}=-900(3+7\mathrm{e}^{-0.25t})^{-2}\left(7(-0.25)\mathrm{e}^{-0.25t}\right)\ \left\{=\dfrac{900(0.25)(7)\mathrm{e}^{-0.25t}}{(3+7\mathrm{e}^{-0.25t})^2}\right\}$	M1	2.1
		A1	1.1b
	$\dfrac{N(300-N)}{1200}=\dfrac{\left(\dfrac{900}{3+7\mathrm{e}^{-0.25t}}\right)\left(300-\dfrac{900}{3+7\mathrm{e}^{-0.25t}}\right)}{1200}$	dM1	2.1
	$\text{LHS}=\dfrac{1575\mathrm{e}^{-0.25t}}{(3+7\mathrm{e}^{-0.25t})^2}$ o.e., $\text{RHS}=\dfrac{900(300(3+7\mathrm{e}^{-0.25t})-900)}{1200(3+7\mathrm{e}^{-0.25t})^2}=\dfrac{1575\mathrm{e}^{-0.25t}}{(3+7\mathrm{e}^{-0.25t})^2}$ o.e. and states hence $\dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{N(300-N)}{1200}$ (or LHS = RHS) *	A1*	1.1b
		(4)	
(c)	Deduces $N=150$ (can be implied)	B1	2.2a
	so $150=\dfrac{900}{3+7\mathrm{e}^{-0.25T}}\Rightarrow \mathrm{e}^{-0.25T}=\dfrac{3}{7}$	M1	3.4
	$T=-4\ln\left(\dfrac{3}{7}\right)$ or $T=$ awrt 3.4 (months)	dM1	1.1b
		A1	1.1b
		(4)	
(d)	either one of 299 or 300	B1	3.4
		(1)	
			(10 marks)

Notes for Question	
(b)	
M1:	Attempts to differentiate using • the chain rule to give $\frac{dN}{dt} = \pm Ae^{-0.25t}(3+7e^{-0.25t})^{-2}$ or $\frac{\pm Ae^{-0.25t}}{(3+7e^{-0.25t})^2}$ o.e. • the quotient rule to give $\frac{dN}{dt} = \frac{(3+7e^{-0.25t})(0) \pm Ae^{-0.25t}}{(3+7e^{-0.25t})^2}$ • implicit differentiation to give $N(3+7e^{-0.25t}) = 900 \Rightarrow (3+7e^{-0.25t})\frac{dN}{dt} \pm ANe^{-0.25t} = 0$, o.e. where $A \neq 0$
Note:	Condone a slip in copying $(3+7e^{-0.25t})$ for the M mark
A1:	A correct differentiation statement
Note:	Implicit differentiation gives $(3+7e^{-0.25t})\frac{dN}{dt} - 1.75Ne^{-0.25t} = 0$
dM1:	**Way 1:** Complete attempt, by eliminating t, to form an equation linking $\frac{dN}{dt}$ and N only **Way 2:** Complete substitution of $N = \frac{900}{3+7e^{-0.25t}}$ into $\frac{dN}{dt} = \frac{N(300-N)}{1200}$
Note:	**Way 1:** e.g. substitutes $3+7e^{-0.25t} = \frac{900}{N}$ and $e^{-0.25t} = \frac{900}{N}$ or substitutes $e^{-0.25t} = \frac{\frac{900}{N}-3}{7}$ into their $\frac{dN}{dt} = ...$ to form an equation linking $\frac{dN}{dt}$ and N
A1*:	**Way 1:** Correct algebra leading to $\frac{dN}{dt} = \frac{N(300-N)}{1200}$ * **Way 2:** See scheme
(c)	
B1:	Deduces or shows that $\frac{dN}{dt}$ is maximised when $N = 150$
M1:	Uses the model $N = \frac{900}{3+7e^{-0.25t}}$ with their $N = 150$ and proceeds as far as $e^{-0.25T} = k,\ k > 0$ or $e^{0.25T} = k,\ k > 0$. Condone $t \equiv T$
dM1:	Correct method of using logarithms to find a value for T. Condone $t \equiv T$
A1:	see scheme
Note:	$\frac{d^2N}{dt^2} = \frac{dN}{dt}\left(\frac{300}{1200} - \frac{2N}{1200}\right) = 0 \Rightarrow N = 150$ is acceptable for B1
Note:	Ignore units for T
Note:	Applying $300 = \frac{900}{3+7e^{-0.25t}} \Rightarrow t = ...$ or $0 = \frac{900}{3+7e^{-0.25t}} \Rightarrow t = ...$ is M0 dM0 A0
Note:	M1 dM1 can only be gained in (c) by using an N value in the range $90 < N < 300$
(d)	
B1:	300 (or accept 299)

Question	Scheme	Marks	AOs
	$N = \dfrac{900}{3+7e^{-0.25t}} = 900(3+7e^{-0.25t})^{-1}, t \in \mathbb{R}, t \geq 0$; $\dfrac{dN}{dt} = \dfrac{N(300-N)}{1200}$		
(b) Way 3	$\displaystyle\int \frac{1}{N(300-N)}\,dN = \int \frac{1}{1200}\,dt$	M1	2.1
	$\displaystyle\int \frac{1}{300}\left(\frac{1}{N} + \frac{1}{300-N}\right)dN = \int \frac{1}{1200}\,dt$ $\dfrac{1}{300}\ln N - \dfrac{1}{300}\ln(300-N) = \dfrac{1}{1200}t \ \{+c\}$	A1	1.1b
	$\{t=0, N=90 \Rightarrow\}\ c = \dfrac{1}{300}\ln(90) - \dfrac{1}{300}\ln(210) \Rightarrow c = \dfrac{1}{300}\ln\left(\dfrac{3}{7}\right)$ $\dfrac{1}{300}\ln N - \dfrac{1}{300}\ln(300-N) = \dfrac{1}{1200}t + \dfrac{1}{300}\ln\left(\dfrac{3}{7}\right)$ $\ln N - \ln(300-N) = \dfrac{1}{4}t + \ln\left(\dfrac{3}{7}\right)$ $\ln\left(\dfrac{N}{300-N}\right) = \dfrac{1}{4}t + \ln\left(\dfrac{3}{7}\right) \Rightarrow \dfrac{N}{300-N} = \dfrac{3}{7}e^{\frac{1}{4}t}$	dM1	2.1
	$7N = 3e^{\frac{1}{4}t}(300-N) \Rightarrow 7N + 3Ne^{\frac{1}{4}t} = 900e^{\frac{1}{4}t}$ $N(7+3e^{\frac{1}{4}t}) = 900e^{\frac{1}{4}t} \Rightarrow N = \dfrac{900e^{\frac{1}{4}t}}{7+3e^{\frac{1}{4}t}} \Rightarrow N = \dfrac{900}{3+7e^{-0.25t}}$ *	A1*	1.1b
		(4)	
(b) Way 4	$N(3+7e^{-0.25t}) = 900 \Rightarrow e^{-0.25t} = \dfrac{1}{7}\left(\dfrac{900}{N} - 3\right) \Rightarrow e^{-0.25t} = \dfrac{900-3N}{7N}$ $\Rightarrow t = -4(\ln(900-3N) - \ln(7N))$	M1	2.1
	$\Rightarrow \dfrac{dt}{dN} = -4\left(\dfrac{-3}{900-3N} - \dfrac{7}{7N}\right)$	A1	1.1b
	$\dfrac{dt}{dN} = 4\left(\dfrac{1}{300-N} + \dfrac{1}{N}\right) \Rightarrow \dfrac{dt}{dN} = 4\left(\dfrac{N+300-N}{N(300-N)}\right)$	dM1	2.1
	$\dfrac{dt}{dN} = \left(\dfrac{1200}{N(300-N)}\right) \Rightarrow \dfrac{dN}{dt} = \dfrac{N(300-N)}{1200}$ *	A1*	1.1b
		(4)	

Notes for Question Continued

(b) Way 3	
M1:	Separates the variables, an attempt to form and apply partial fractions and integrates to give ln terms $= kt\ \{+c\},\ k \neq 0$, with or without a constant of integration c
A1:	$\dfrac{1}{300}\ln N - \dfrac{1}{300}\ln(300-N) = \dfrac{1}{1200}t\ \{+c\}$ or equivalent with or without a constant of integration c
dM1:	Uses $t=0$, $N=90$ to find their constant of integration and obtains an expression of the form $\lambda e^{\frac{1}{4}t} = f(N);\ \lambda \neq 0$ or $\lambda e^{-\frac{1}{4}t} = f(N);\ \lambda \neq 0$
A1*:	Correct manipulation leading to $N = \dfrac{900}{3+7e^{-0.25t}}$ *
(b) Way 4	
M1:	Valid attempt to make t the subject, followed by an attempt to find two ln derivatives, condoning sign errors and constant errors.
A1:	$\dfrac{dt}{dN} = -4\left(\dfrac{-3}{900-3N} - \dfrac{7}{7N}\right)$ or equivalent
dM1:	Forms a common denominator to combine their fractions
A1*:	Correct algebra leading to $\dfrac{dN}{dt} = \dfrac{N(300-N)}{1200}$ *

Q3.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x+1)^2\times(10x+10)-(5x^2+10x)\times 2(x+1)}{(x+1)^4}$	M1	This mark is given for an attempt to differentiate the expression for y
		A1	This mark is given for correctly differentiating the expression for y
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x+1)\times(10x+10)-(5x^2+10x)\times 2}{(x+1)^3}$	M1	This mark sis given for cancelling the expression through by $(x + 1)$
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{10}{(x+1)^3}$	A1	This mark is given for a fully correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$
(b)	If $A > 0$ and $n = 1, 3$ then $x < -1$	B1	This mark is given for deducing that $\frac{\mathrm{d}y}{\mathrm{d}x} < 0 \Rightarrow x < -1$.

Q4.

Question	Scheme	Marks	AOs
(a)	$\ln x \to \frac{1}{x}$	B1	1.1a
	Method to differentiate $\frac{4x^2+x}{2\sqrt{x}}$ – see notes	M1	1.1b
	E.g. $2\times\frac{3}{2}x^{\frac{1}{2}}+\frac{1}{2}\times\frac{1}{2}x^{-\frac{1}{2}}$	A1	1.1b
	$\frac{\mathrm{d}y}{\mathrm{d}x}=3\sqrt{x}+\frac{1}{4\sqrt{x}}-\frac{4}{x}=\frac{12x^2+x-16\sqrt{x}}{4x\sqrt{x}}$ *	A1*	2.1
		(4)	
(b)	$12x^2+x-16\sqrt{x}=0 \Rightarrow 12x^{\frac{3}{2}}+x^{\frac{1}{2}}-16=0$	M1	1.1b
	E.g. $12x^{\frac{3}{2}}=16-\sqrt{x}$	dM1	1.1b
	$x^{\frac{3}{2}}=\frac{4}{3}-\frac{\sqrt{x}}{12} \Rightarrow x=\left(\frac{4}{3}-\frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}$ *	A1*	2.1
		(3)	
(c)	$x_2=\sqrt[3]{\left(\frac{4}{3}-\frac{\sqrt{2}}{12}\right)^2}$	M1	1.1b
	x_2 = awrt 1.13894	A1	1.1b
	$x = 1.15650$	A1	2.2a
		(3)	
			(10 marks)

Notes:

(a)

B1: Differentiates $\ln x \to \frac{1}{x}$ seen or implied

M1: Correct method to differentiate $\frac{4x^2+x}{2\sqrt{x}}$:

Look for $\frac{4x^2+x}{2\sqrt{x}} \to ...x^{\frac{3}{2}} + ...x^{\frac{1}{2}}$ being then differentiated to $Px^{\frac{1}{2}} + ...$ or $... + Qx^{-\frac{1}{2}}$

Alternatively uses the quotient rule on $\frac{4x^2+x}{2\sqrt{x}}$.

Condone slips but if rule is not quoted expect $\left(\frac{dy}{dx}\right) = \frac{2\sqrt{x}(Ax+B)-(4x^2+x)Cx^{-\frac{1}{2}}}{(2\sqrt{x})^2}(A, B, C > 0)$

But a correct rule may be implied by their u, v, u', v' followed by applying $\frac{vu'-uv'}{v^2}$ etc.

Alternatively uses the product rule on $(4x^2+x)(2\sqrt{x})^{-1}$

Condone slips but expect $\left(\frac{dy}{dx}\right) = Ax^{-\frac{1}{2}}(Bx+C) + D(4x^2+x)x^{-\frac{3}{2}}(A, B, C > 0)$

In general condone missing brackets for the M mark. If they quote $u = 4x^2 + x$ and $v = 2\sqrt{x}$ and don't make the differentiation easier, they can be awarded this mark for applying the correct rule. Also allow this mark if they quote the correct quotient rule but only have v rather than v^2 in the denominator.

A1: Correct differentiation of $\frac{4x^2+x}{2\sqrt{x}}$ although may not be simplified.

Examples: $\left(\frac{dy}{dx}\right) = \frac{2\sqrt{x}(8x+1)-(4x^2+x)x^{-\frac{1}{2}}}{(2\sqrt{x})^2}$, $\frac{1}{2}x^{-\frac{1}{2}}(8x+1) - \frac{1}{4}(4x^2+x)x^{-\frac{3}{2}}$, $2\times\frac{3}{2}x^{\frac{1}{2}} + \frac{1}{2}\times\frac{1}{2}x^{-\frac{1}{2}}$

A1*: Obtains $\frac{dy}{dx} = \frac{12x^2+x-16\sqrt{x}}{4x\sqrt{x}}$ via $3\sqrt{x} + \frac{1}{4\sqrt{x}} - \frac{4}{x}$ or a correct application of the quotient or product rule and with sufficient working shown to reach the printed answer.

There must be no errors e.g. missing brackets.

(b)

M1: Sets $12x^2 + x - 16\sqrt{x} = 0$ and divides by $\sqrt{x}$ or equivalent e.g. divides by x and multiplies by $\sqrt{x}$

dM1: Makes the term in $x^{\frac{3}{2}}$ the subject of the formula

A1*: A correct and rigorous argument leading to the given solution.

Alternative - working backwards:

$$x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12}\right)^{\frac{2}{3}} \Rightarrow x^{\frac{3}{2}} = \frac{4}{3} - \frac{\sqrt{x}}{12} \Rightarrow 12x^{\frac{3}{2}} = 16 - \sqrt{x} \Rightarrow 12x^2 = 16\sqrt{x} - x \Rightarrow 12x^2 - 16\sqrt{x} + x = 0$$

M1: For raising to power of 3/2 both sides. dM1: Multiplies through by $\sqrt{x}$. A1: Achieves printed answer and makes a minimal comment e.g. tick, #, QED, true etc.

(c)

M1: Attempts to use the iterative formula with $x_1 = 2$. This is implied by sight of $x_2 = \left(\frac{4}{3} - \frac{\sqrt{2}}{12}\right)^{\frac{2}{3}}$ or awrt 1.14

A1: x_2 = awrt 1.13894

A1: Deduces that $x = 1.15650$

Q5.

Question	Scheme	Marks	AOs
(a)	$k = e^2$ or $x \neq e^2$	B1	2.2a
		(1)	
(b)	$g'(x) = \dfrac{(\ln x - 2)\times\frac{3}{x} - (3\ln x - 7)\times\frac{1}{x}}{(\ln x - 2)^2} = \dfrac{1}{x(\ln x - 2)^2}$ or $g'(x) = \dfrac{d}{dx}\left(3 - (\ln(x) - 2)^{-1}\right) = (\ln x - 2)^{-2}\times\dfrac{1}{x} = \dfrac{1}{x(\ln x - 2)^2}$ or $g'(x) = (\ln x - 2)^{-1}\times\dfrac{3}{x} - (3\ln x - 7)(\ln x - 2)^{-2}\times\dfrac{1}{x} = \dfrac{1}{x(\ln x - 2)^2}$	M1 A1	1.1b 2.1
	As $x > 0$ (or $1/x > 0$) **AND** $\ln x - 2$ is squared so $g'(x) > 0$	A1cso	2.4
		(3)	
(c)	Attempts to solve either $3\ln x - 7 \ldots 0$ or $\ln x - 2 \ldots 0$ or $3\ln a - 7 \ldots 0$ or $\ln a - 2 \ldots 0$ where … is "=" or ">" to reach a value for x or a but may be seen as an inequality e.g. $x > \ldots$ or $a > \ldots$	M1	3.1a
	$0 < a < e^2, \quad a > e^{\frac{7}{3}}$	A1	2.2a
		(2)	
			(6 marks)

Notes:

(a)

B1: Deduces $k = e^2$ or $x \neq e^2$ Condone $k =$ awrt 7.39 or $x \neq$ awrt 7.39

(b)

M1: Attempts to differentiate via the quotient rule and with $\ln x \to \frac{1}{x}$ so allow for:

$$\frac{d}{dx}(g(x)) = \frac{(\ln x - 2)\times\frac{\alpha}{x} - (3\ln x - 7)\times\frac{\beta}{x}}{(\ln x - 2)^2}, \quad \beta > 0$$

But a correct rule may be implied by their u, v, u', v' followed by applying $\frac{vu' - uv'}{v^2}$ etc.

Alternatively attempts to write $g(x) = \dfrac{3\ln(x) - 7}{\ln(x) - 2} = 3 - (\ln(x) - 2)^{-1}$ and attempts the chain rule so allow for:

$$3 - (\ln(x) - 2)^{-1} \to (\ln(x) - 2)^{-2}\times\frac{\alpha}{x}$$

Alternatively writes $g(x) = (3\ln(x) - 7)(\ln(x) - 2)^{-1}$ and attempts the product rule so allow for:

$$g'(x) = (\ln x - 2)^{-1}\times\frac{\alpha}{x} - (3\ln x - 7)(\ln x - 2)^{-2}\times\frac{\beta}{x}$$

In general condone missing brackets for the M mark. E.g. if they quote $u = 3\ln x - 7$ and $v = \ln x - 2$ and don't make the differentiation easier, they can be awarded this mark for applying the correct rule. Also allow this mark if they quote the correct quotient rule but only have v rather than v^2 in the denominator.

A1: $\dfrac{1}{x(\ln x - 2)^2}$ **Allow** $\dfrac{\frac{1}{x}}{(\ln x - 2)^2}$ **i.e. we need to see the numerator simplified to $1/x$**

Note that some candidates establish the correct numerator and correct denominator independently and provided they obtain the correct expressions, this mark can be awarded.
But allow a correctly expanded denominator.

A1cso: States that as $x > 0$ **AND** $\ln x - 2$ is squared so $g'(x) > 0$

(c)

M1: Attempts to solve either $3\ln x - 7 = 0$ or $\ln x - 2 = 0$ or using inequalities e.g. $3\ln x - 7 > 0$

A1: $0 < a < e^2,\ a > e^{\frac{7}{3}}$

Q6.

Question	Scheme	Marks	AOs
	$y = \dfrac{x-4}{2+\sqrt{x}} \Rightarrow \dfrac{dy}{dx} = \dfrac{2+\sqrt{x}-(x-4)\frac{1}{2}x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2}$	M1 A1	2.1 1.1b
	$= \dfrac{2+\sqrt{x}-(x-4)\frac{1}{2}x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2} = \dfrac{2+\sqrt{x}-\frac{1}{2}\sqrt{x}+2x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2} = \dfrac{2\sqrt{x}+\frac{1}{2}x+2}{\sqrt{x}\left(2+\sqrt{x}\right)^2}$	M1	1.1b
	$= \dfrac{x+4\sqrt{x}+4}{2\sqrt{x}\left(2+\sqrt{x}\right)^2} = \dfrac{\left(2+\sqrt{x}\right)^2}{2\sqrt{x}\left(2+\sqrt{x}\right)^2} = \dfrac{1}{2\sqrt{x}}$	A1	2.1
		(4)	
			(4 marks)
Notes			

M1: Attempts to use a correct rule e.g. quotient or product (& chain) rule to achieve the following forms

Quotient : $\dfrac{\alpha\left(2+\sqrt{x}\right)-\beta(x-4)x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2}$ but be tolerant of attempts where the $\left(2+\sqrt{x}\right)^2$ has been incorrectly expanded

Product: $\alpha\left(2+\sqrt{x}\right)^{-1} + \beta x^{-\frac{1}{2}}(x-4)\left(2+\sqrt{x}\right)^{-2}$

Alternatively with $t = \sqrt{x}$, $y = \dfrac{t^2-4}{2+t} \Rightarrow \dfrac{dy}{dx} = \dfrac{dy}{dt}\times\dfrac{dt}{dx} = \dfrac{2t(2+t)-\left(t^2-4\right)}{(2+t)^2}\times\dfrac{1}{2}x^{-\frac{1}{2}}$ with same rules

A1: Correct derivative in any form. Must be in terms of a single variable (which could be t)

M1: Following a correct attempt at differentiation, it is scored for multiplying both numerator and denominator by $\sqrt{x}$ and collecting terms to form a single fraction. It can also be scored from $\dfrac{uv'-vu'}{v^2}$

For the $t = \sqrt{x}$, look for an attempt to simplify $\dfrac{t^2+4t+4}{(2+t)^2}\times\dfrac{1}{2t}$

A1: Correct expression showing all key steps with no errors or omissions. $\dfrac{dy}{dx}$ must be seen at least once

Question	Scheme	Marks	AOs
	$y = \dfrac{x-4}{2+\sqrt{x}} \Rightarrow y = \dfrac{\left(\sqrt{x}+2\right)\left(\sqrt{x}-2\right)}{2+\sqrt{x}} = \sqrt{x}-2$	M1 A1	2.1 1.1b
	$\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x}}$	M1 A1	1.1b 2.1
		(4)	
			(4 marks)
Notes			

M1: Attempts to use difference of two squares. Can also be scored using

$t=\sqrt{x} \Rightarrow y=\dfrac{t^2-4}{t+2} \Rightarrow y=\dfrac{(t+2)(t-2)}{t+2}$

A1: $y=\sqrt{x}-2$ or $y=t-2$

M1: Attempts to differentiate an expression of the form $y=\sqrt{x}+b$

A1: Correct expression showing all key steps with no errors or omissions. $\dfrac{dy}{dx}$ must be seen at least once

Q7.

Question	Scheme	Marks	AOs
(a)	Attempts to differentiate using the quotient rule or otherwise	M1	2.1
	$f'(x)=\dfrac{e^{\sqrt{2}x-1}\times 8\cos 2x-4\sin 2x\times\sqrt{2}e^{\sqrt{2}x-1}}{\left(e^{\sqrt{2}x-1}\right)^2}$	A1	1.1b
	Sets $f'(x)=0$ and divides/ factorises out the $e^{\sqrt{2}x-1}$ terms	M1	2.1
	Proceeds via $\dfrac{\sin 2x}{\cos 2x}=\dfrac{8}{4\sqrt{2}}$ to $\Rightarrow \tan 2x=\sqrt{2}$ *	A1*	1.1b
		(4)	
(b)	(i) Solves $\tan 4x=\sqrt{2}$ and attempts to find the 2nd solution	M1	3.1a
	$x=1.02$	A1	1.1b
	(ii) Solves $\tan 2x=\sqrt{2}$ and attempts to find the 1st solution	M1	3.1a
	$x=0.478$	A1	1.1b
		(4)	
			(8 marks)

Notes:
(a)
M1: Attempts to differentiate by using the quotient rule with $u = 4\sin 2x$ and $v = e^{\sqrt{2}x-1}$ or alternatively uses the product rule with $u = 4\sin 2x$ and $v = e^{1-\sqrt{2}x}$
A1: For achieving a correct $f'(x)$. For the product rule $f'(x) = e^{1-\sqrt{2}x} \times 8\cos 2x + 4\sin 2x \times -\sqrt{2}e^{1-\sqrt{2}x}$
M1: This is scored for cancelling/ factorising out the exponential term. Look for an equation in just $\cos 2x$ and $\sin 2x$
A1*: Proceeds to $\tan 2x = \sqrt{2}$. This is a given answer.
(b) (i)
M1: Solves $\tan 4x = \sqrt{2}$ attempts to find the 2nd solution. Look for $x = \dfrac{\pi + \arctan\sqrt{2}}{4}$
Alternatively finds the 2nd solution of $\tan 2x = \sqrt{2}$ and attempts to divide by 2
A1: Allow awrt $x = 1.02$. The correct answer, with no incorrect working scores both marks
(b)(ii)
M1: Solves $\tan 2x = \sqrt{2}$ attempts to find the 1st solution. Look for $x = \dfrac{\arctan\sqrt{2}}{2}$
A1: Allow awrt $x = 0.478$. The correct answer, with no incorrect working scores both marks

Q8.

Question	Scheme	Marks	AOs
	Attempts the product and chain rule on $y = x(2x+1)^4$	M1	2.1
	$\dfrac{dy}{dx} = (2x+1)^4 + 8x(2x+1)^3$	A1	1.1b
	Takes out a common factor $\dfrac{dy}{dx} = (2x+1)^3\{(2x+1)+8x\}$	M1	1.1b
	$\dfrac{dy}{dx} = (2x+1)^3(10x+1) \Rightarrow n = 3, A = 10, B = 1$	A1	1.1b
		(4 marks)	

Notes:
M1: Applies the product rule to reach $\dfrac{dy}{dx} = (2x+1)^4 + Bx(2x+1)^3$
A1: $\dfrac{dy}{dx} = (2x+1)^4 + 8x(2x+1)^3$
M1: Takes out a common factor of $(2x+1)^3$
A1: The form of this answer is given. Look for $\dfrac{dy}{dx} = (2x+1)^3(10x+1) \Rightarrow n = 3, A = 10, B = 1$

Q9.

Question	Scheme	Marks	AOs
	$\dfrac{1+11x-6x^2}{(x-3)(1-2x)} \equiv A+\dfrac{B}{(x-3)}+\dfrac{C}{(1-2x)}$		
(a) Way 1	$1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3) \Rightarrow B=..., C=...$	M1	2.1
	$A=3$	B1	1.1b
	Uses substitution or compares terms to find either $B=...$ or $C=...$	M1	1.1b
	$B=4$ and $C=-2$ which have been found using a correct identity	A1	1.1b
		(4)	
(a) Way 2	{long division gives} $\dfrac{1+11x-6x^2}{(x-3)(1-2x)} \equiv 3+\dfrac{-10x+10}{(x-3)(1-2x)}$		
	$-10x+10 \equiv B(1-2x)+C(x-3) \Rightarrow B=..., C=...$	M1	2.1
	$A=3$	B1	1.1b
	Uses substitution or compares terms to find either $B=...$ or $C=...$	M1	1.1b
	$B=4$ and $C=-2$ which have been found using $-10x+10 \equiv B(1-2x)+C(x-3)$	A1	1.1b
		(4)	
(b)	$\mathrm{f}(x)=3+\dfrac{4}{(x-3)}-\dfrac{2}{(1-2x)}$ $\{=3+4(x-3)^{-1}-2(1-2x)^{-1}\}$; $x>3$		
	$\mathrm{f}'(x)=-4(x-3)^{-2}-4(1-2x)^{-2}$ $\left\{=-\dfrac{4}{(x-3)^2}-\dfrac{4}{(1-2x)^2}\right\}$	M1 A1ft	2.1 1.1b
	Correct $\mathrm{f}'(x)$ and as $(x-3)^2>0$ and $(1-2x)^2>0$, then $\mathrm{f}'(x)=-(+\text{ve})-(+\text{ve})<0$, so $\mathrm{f}(x)$ is a decreasing function	A1	2.4
		(3)	
		(7 marks)	

Notes for Question	
(a)	
M1:	**Way 1:** Uses a correct identity $1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3)$ in a complete method to find values for B and C. **Note:** Allow one slip in copying $1+11x-6x^2$ **Way 2:** Uses a correct identity $-10x+10 \equiv B(1-2x)+C(x-3)$ (which has been found from long division) in a complete method to find values for B and C
B1:	$A=3$
M1:	Attempts to find the value of either B or C from their identity This can be achieved by *either* substituting values into their identity *or* by comparing coefficients and solving the resulting equations simultaneously
A1:	See scheme
Note:	**Way 1:** Comparing terms: $x^2: -6=-2A;\quad x: 11=7A-2B+C;\quad \text{constant}: 1=-3A+B-3C$ **Way 1:** Substituting: $x=3: -20=-5B \Rightarrow B=4$; $x=\frac{1}{2}: 5=-\frac{5}{2}C \Rightarrow C=-2$
Note:	**Way 2:** Comparing terms: $x: -10=-2B+C;\quad \text{constant}: 10=B-3C$ **Way 2:** Substituting: $x=3: -20=-5B \Rightarrow B=4$; $x=\frac{1}{2}: 5=-\frac{5}{2}C \Rightarrow C=-2$
Note:	$A=3, B=4, C=-2$ from no working scores M1B1M1A1
Note:	The final A1 mark is effectively dependent upon both M marks

Notes for Question Continued	
(a) ctd	
Note:	Writing $1+11x-6x^2 \equiv B(1-2x)+C(x-3) \Rightarrow B=4,\ C=-2$ will get 1st M0, 2nd M1, 1st A0
Note:	Way 1: You can imply a correct identity $1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3)$ from seeing $\dfrac{1+11x-6x^2}{(x-3)(1-2x)} \equiv \dfrac{A(1-2x)(x-3)+B(1-2x)+C(x-3)}{(x-3)(1-2x)}$
Note:	Way 2: You can imply a correct identity $-10x+10 \equiv B(1-2x)+C(x-3)$ from seeing $\dfrac{-10x+10}{(x-3)(1-2x)} \equiv \dfrac{B(1-2x)+C(x-3)}{(x-3)(1-2x)}$
(b)	
M1:	Differentiates to give $\{\mathrm{f}'(x)=\}\ \pm\lambda(x-3)^{-2} \pm \mu(1-2x)^{-2};\ \lambda,\ \mu \neq 0$
A1ft:	$\mathrm{f}'(x) = -4(x-3)^{-2} - 4(1-2x)^{-2}$, which can be simplified or un-simplified
Note:	Allow A1ft for $\mathrm{f}'(x) = -(\text{their } B)(x-3)^{-2} + (2)(\text{their } C)(1-2x)^{-2}$; (their B), (their C) $\neq 0$
A1:	$\mathrm{f}'(x) = -4(x-3)^{-2} - 4(1-2x)^{-2}$ or $\mathrm{f}'(x) = -\dfrac{4}{(x-3)^2} - \dfrac{4}{(1-2x)^2}$ and a correct explanation e.g. $\mathrm{f}'(x) = -(+\text{ve}) - (+\text{ve}) < 0$, so $\mathrm{f}(x)$ is a decreasing {function}
Note:	The final A mark can be scored in part (b) from an incorrect $A = \ldots$ or from $A = 0$ or no value of A found in part (a)

Notes for Question Continued - Alternatives	
(a)	
Note:	Be aware of the following alternative solutions, by initially dividing by "$(x-3)$" or "$(1-2x)$" • $\dfrac{1+11x-6x^2}{"(x-3)"(1-2x)} \equiv \dfrac{-6x-7}{(1-2x)} - \dfrac{20}{(x-3)(1-2x)} \equiv 3 - \dfrac{10}{(1-2x)} - \dfrac{20}{(x-3)(1-2x)}$ $\dfrac{20}{(x-3)(1-2x)} \equiv \dfrac{D}{(x-3)} + \dfrac{E}{(1-2x)} \Rightarrow 20 \equiv D(1-2x)+E(x-3) \Rightarrow D=-4,\ E=-8$ $\Rightarrow 3 - \dfrac{10}{(1-2x)} - \left(\dfrac{-4}{(x-3)} + \dfrac{-8}{(1-2x)}\right) \equiv 3 + \dfrac{4}{(x-3)} - \dfrac{2}{(1-2x)};\ A=3,\ B=4,\ C=-2$ • $\dfrac{1+11x-6x^2}{(x-3)"(1-2x)"} \equiv \dfrac{3x-4}{(x-3)} + \dfrac{5}{(x-3)(1-2x)} \equiv 3 + \dfrac{5}{(x-3)} + \dfrac{5}{(x-3)(1-2x)}$ $\dfrac{5}{(x-3)(1-2x)} \equiv \dfrac{D}{(x-3)} + \dfrac{E}{(1-2x)} \Rightarrow 5 \equiv D(1-2x)+E(x-3) \Rightarrow D=-1,\ E=-2$ $\Rightarrow 3 + \dfrac{5}{(x-3)} + \left(\dfrac{-1}{(x-3)} + \dfrac{-2}{(1-2x)}\right) \equiv 3 + \dfrac{4}{(x-3)} - \dfrac{2}{(1-2x)};\ A=3,\ B=4,\ C=-2$
(b)	

	Alternative Method 1:		
	$\mathrm{f}(x) = \dfrac{1+11x-6x^2}{(x-3)(1-2x)},\ x>3 \Rightarrow \mathrm{f}(x) = \dfrac{1+11x-6x^2}{-2x^2+7x-3};\ \begin{cases} u = 1+11x-6x^2 & v = -2x^2+7x-3 \\ u' = 11-12x & v' = -4x+7 \end{cases}$		
	$\mathrm{f}'(x) = \dfrac{(-2x^2+7x-3)(11-12x)-(1+11x-6x^2)(-4x+7)}{(-2x^2+7x-3)^2}$	Uses quotient rule to find $\mathrm{f}'(x)$	M1
		Correct differentiation	A1
	$\mathrm{f}'(x) = \dfrac{-20((x-1)^2+1)}{(-2x^2+7x-3)^2}$ and a correct explanation, e.g. $\mathrm{f}'(x) = -\dfrac{(+\text{ve})}{(+\text{ve})} < 0$, so $\mathrm{f}(x)$ is a decreasing {function}		A1
	Alternative Method 2: Allow M1A1A1 for the following solution: Given $\mathrm{f}(x) = 3 + \dfrac{4}{(x-3)} - \dfrac{2}{(1-2x)} = 3 + \dfrac{4}{(x-3)} + \dfrac{2}{(2x-1)}$ as $\dfrac{4}{(x-3)}$ decreases when $x>3$ and $\dfrac{2}{(2x-1)}$ decreases when $x>3$ then $\mathrm{f}(x)$ is a decreasing {function}		